Tentamen Statistiek voor KI/Inf

Tuesday 31 October 2006 - Open book tentamen

All books and all calculators allowed, cell phones and laptops not allowed.

1. Suppose that $f_Y(y) = c(4-y)^2$ for $0 \leq y \leq 4$ is the density of a random variable Y.

   (a) Determine c so that this defines a proper density.

   (b) Compute the expected value $E(Y)$, $E(Y^2)$, and the variance.

   (c) Compute the cdf (the cumulative distribution function).

   (d) Suppose that 6 random variables are drawn from this distribution. What is the probability that precisely 4 of these random variables have values in the interval $[0, 3]$?

2. Consider the exponential density $f_Y(x) = 3e^{-3x}$ for $0 < x < \infty$.

   (a) Compute the expected value of Y.

   (b) Compute the variance.

   (c) Give the pdf (probability distribution function) of the sum of two independent random variables with this distribution.

   (d) What would be the answer for a sum of 20 independent random variables?

3. **(a)** Based on the random sample $6.1, 1.9, 2.0, 0.3, 5, 2.1$ use the method of maximum likelihood to estimate the parameter θ in the uniform pdf $f_Y(y) = \frac{1}{\theta}$ for $0 \leq y \leq \theta$.

   (b) Suppose the random sample in Part (a) represents the two-parameter uniform pdf $f_Y(y, \theta_1, \theta_2) = \frac{1}{\theta_2 - \theta_1}$ for $\theta_1 \leq y \leq \theta_2$.

   Find the maximum likelihood estimates for θ_1 and θ_2

   (c) Compute the expected values of the maximum likelihood estimators from Part (b) if θ_2 and θ_1 are the true parameters.

4. Consider a normal random sample $X_1, \ldots, X_n$ with expectation μ and variance σ^2, where the variance σ^2 is unknown.

   (a) Suppose that a test for the mean μ is made, based on one random sample. Consider the following statements:

   (S1) We reject the null Hypothesis $\mu = 0$ against the alternative $\mu < 0$ to the level $\alpha = 0.05$.

   (S2) We reject the null Hypothesis $\mu = 0$ against the alternative $\mu \neq 0$ to the level $\alpha = 0.05$.

Is it true that (S2) follows from (S1)? Is it true that (S1) follows from (S2)?

(b) Now two different samples from this population are taken. A 90-percent confidence interval for μ is constructed with the first sample, and a 95-percent confidence interval is constructed with the second. Will the 95-percent confidence interval necessarily be longer than the 90-percent confidence interval? Explain!

5. **(a)** Give the definition of an unbiased estimator!

   (b) Is the maximum likelihood estimator always unbiased? Explain or give a counter-example!

   (c) Suppose that $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ are independent random samples from normal distributions with the same σ^2. Is the pooled variance

$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}$$

   an unbiased estimator for σ^2?